Topic 7: Multiplying and Dividing Rational Numbers

Problem 7.1

A. Do the following calculations and use the results to help you answer the questions that follow.

Multiplication	Division
$12 \times 3 =$	$12 \div 3 =$
$18 \times 1\frac{1}{3} =$	$18 \div 1\frac{1}{3} =$
$12 \times \frac{1}{3} =$	$12 \div \frac{1}{3} =$
$18 \times \frac{1}{9} =$	$18 \div \frac{1}{9} =$

1. When you multiply a number by a value *greater than 1*, how can you describe the size of the product as compared to the number? Give an example from the table.

2. When you multiply a number by a value *between 0 and 1,* how can you describe the size of the product as compared to the number? Give an example from the table.

3. When you divide a number by a value *greater than 1,* how can you describe the size of the quotient as compared to the number? Give an example from the table.

4. When you divide a number by a value *between 0 and 1,* how can you describe the size of the quotient compared to the number? Give an example from the table.

Problem 7.2

A. Dana used rounding to get 180 as an estimated product for 59×3. How did she round the factors?

B. Natalya used rounding to get 400 as an estimated product for 41.7×9.6. How did she round the factors?

C. In order to estimate the quotient $573 \div 28.2$, Sam rounded the numbers to the nearest whole numbers, 573 and 28.

   1. Do you agree with Sam's rounding? Explain.

   2. Are there better numbers to use when rounding? If so, what is your suggestion?

   3. What estimated quotient do you get with your suggested numbers?

Problem 7.3

A. A **benchmark fraction** is a fraction that is easy to calculate with.

    1. To estimate the product $60 \times \frac{5}{9}$ what benchmark fraction could you use instead of $\frac{5}{9}$?

    2. Estimate the product using the benchmark fraction.

B. Rafiq wants to estimate $23.4 \times \frac{9}{30}$.

    1. What benchmark fraction is close to $\frac{9}{30}$?

    2. If Rafiq uses that benchmark fraction, what is the best way to round 23.4 to make it a compatible number?

    3. Using the benchmark fraction and the compatible number, what estimated product will Rafiq get?

Exercises

1. Predict whether each product or quotient is greater or less than the first factor in the problem.

    a. $129 \div 0.57$ **b.** $\frac{1}{3} \times \frac{1}{8}$ **c.** 0.8×0.64 **d.** $306 \times \frac{12}{7}$

    e. $3\frac{4}{5} \div 4\frac{3}{8}$ **f.** $\frac{9}{4} \div \frac{2}{5}$ **g.** 1.2×63.9 **h.** $72 \div 21.9$

2. Estimate each product or quotient.

    a. 24.9×9.3 **b.** $11.4 \div 6.6$ **c.** $24 \times \frac{1}{13}$ **d.** $80 \div \frac{15}{7}$

    e. $\frac{43}{119} \times 33$ **f.** 0.85×7.3 **g.** $81 \div \frac{89}{9}$ **h.** $72.2 \div 25.3$

3. Estimate $21.8 \div 8$ twice, using a different set of compatible numbers each time. Determine which is the more accurate estimate by calculating the actual quotient and comparing that with your estimated quotients.

4. Hector says to estimate the quotient for $43 \div 13$ by using $40 \div 10$. Rachel says to use $45 \div 15$ and that her estimate will be more accurate than Hector's.

    a. Estimate using Hector's method of rounding.

    b. Estimate using Rachel's method of rounding.

    c. Which estimate will be more accurate? Give your reasoning.

    d. Check your prediction by calculating the exact value of the quotient.

Topic 7: Multiplying and Dividing Rational Numbers

Mathematical Goals

- Relate the size of a number used for multiplying or dividing to the size of the product or quotient, respectively.
- Estimate the results of multiplying and dividing rational numbers.

Guided Instruction

Vocabulary
- benchmark fraction

Having some sense of the dimensions of a product or quotient enables a student to judge the appropriateness of a proposed calculation for a word problem. Additionally, comparing a completed calculation with an estimate provides a check on the reasonableness of the results of the calculation.

One way to anticipate the result of a multiplication operation involving positive numbers is to determine whether one of the factors is less than or greater than 1. For a division operation, the question is whether the divisor is less than or greater than 1. This type of analysis will reveal whether the product or quotient is greater or less than the other number in the calculation.

Some students will always tend to round to a multiple of ten. However, a student who knows the perfect squares to 225 may use $22.5 \div 1.5 = 15$ as an estimate for $22.6 \div 1.48$, and should be encouraged to do so.

After Problem 7.2, ask:

- *How can choosing which way to round the numbers in a multiplication problem affect the estimated product as compared to the actual product?* (If you round both numbers up, the estimated product will be higher than the actual product. If you round both numbers down, the estimated product will be less than the actual product.)

- *Does it make a difference whether you round the dividend and the divisor in the same direction?* (Yes. Rounding the dividend up and the divisor down gives an estimated quotient that is higher than the actual quotient. Rounding the dividend down and the divisor up gives an estimated quotient that is lower than the actual quotient.)

After Problem 7.3, ask:

- *How is substituting a benchmark fraction for an actual fraction similar to rounding the actual fraction?* (They both involve choosing a convenient value that is close to an actual value.)

You will find additional work on rational number operations in the grade 6 unit *Bits and Pieces II*.

Assignment Guide for Topic 7

Core 1–4

Answers to Topic 7

Problem 7.1

A.

Multiplication	Division
$12 \times 3 = 36$	$12 \div 3 = 4$
$18 \times 1\frac{1}{3} = 24$	$18 \div 1\frac{1}{3} = \frac{27}{2}$
$12 \times \frac{1}{3} = 4$	$12 \div \frac{1}{3} = 36$
$18 \times \frac{1}{9} = 2$	$18 \div \frac{1}{9} = 162$

1. The product is greater than the number; $12 \times 3 = 36$

2. The product is less than the number; $12 \times \frac{1}{3} = 4$

3. The quotient is less than the number; $12 \div 3 = 4$

4. The quotient is greater than the number; $12 \times \frac{1}{3} = 36$

Problem 7.2

A. Dana rounded 59 to 60.

B. Natalya rounded 41.7 to 40, and 9.6 to 10.

C. 1. No, 573 and 28 are not compatible.

2. Use 600 and 30.

3. 20

Problem 7.3

A. 1. $\frac{1}{2}$

2. 30

B. 1. $\frac{1}{3}$

2. Round to 24.

3. 8

Exercises

1. a. greater than **b.** less than

c. less than **d.** greater than

e. less than **f.** greater than

g. greater than **h.** less than

2. Answers may vary. Samples:

a. 250 **b.** 2 **c.** 2 **d.** 40

e. 11 **f.** 7 **g.** 8 **h.** 3

3. Answers may vary. Sample: Estimates $20 \div 10 = 2$ and $21 \div 7 = 3$. The actual quotient is 2.73. The second estimate is more accurate than the first.

4. a. 4

b. 3

c. Answers may vary. Sample: Rachel's, because her rounded numbers are just 2 away from the actual numbers, while Hector's are 3 away.

d. The actual quotient is 3.31.